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On collinearization of quarks in the quark-gluon decays of heavy orthoquarkonia

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Abstract

The decay of heavy orthoquarkonium into quark-antiquark pair and two gluons is considered. The differential probability of the decay in the tree approximation is calculated and the probability distributions of quarks and gluons are obtained. The collinear enhancement of the bottomonium decay is shown to take place at the $u\bar{u}$–, $d\bar{d}$ and $s\bar{s}$– pair production and to be absent at the $c\bar{c}$– production. Some other peculiarities of the considered decays are discussed.

The decays of heavy quarkonia give the useful information on the dynamics of quarks and gluons and on the processes of jet production of hadrons. In particular the many-particle decays such as $^1S_0 \to$ 3g, q$\bar{\text{q}}$g and $^3S_1 \to$ 4g, q$\bar{\text{q}}$gg are of great interest as the processes giving the immediate information on q$\bar{\text{q}}$g– and 3g– interactions. The nonabelian nature of ggg-interaction manifests itself, for example, in the distribution in invariant masses of two particles[1,2], as the acomplanarity of four particle decays[3], as the collinearization of the gluons[4] and as some other effects. It is worth noting that the four particle decays of heavy orthoquarkonia are more available for experimental investigations because of rather great probability of direct production of orthoquarkonia in e^+e^-– and $p\bar{p}$– collisions. By this reason the detail theoretical analysis of these decays is interesting in order to find the optimal conditions for their experimental observation.

In this work we consider the decay of heavy orthoquarkonium with spin-parity $J^{PC} = 1^{--}$ into quark-antiquark pair and two gluons. The amplitude and differential probability of this process in tree approximation and some angle and energy distributions of quarks and gluons are obtained and discussed.

The process $^3S_1(Q\bar{Q}) \to$ q$\bar{\text{q}}$gg is described in the tree approximation by six diagrams shown in fig.1. The amplitude of this process with the neglect of the relative momentum of the heavy initial quarks may be presented in the form:

$$M_{ab}^{\alpha\beta}(^3S_1(Q\bar{Q}) \to q\bar{q}gg) = \frac{d_{abc}(t_c)_{\alpha\beta}}{4\sqrt{N_c}} \, \frac{g_{st}^4\psi(0)}{4\sqrt{\varepsilon_1\varepsilon_2\omega_1\omega_2}} \, \frac{J_\mu j_\mu}{p^2(pk)(Pk_1)(Pk_2)}, \qquad (1)$$

$$\begin{aligned} J_\mu &= \bar{u}_Q(-P/2)\{\hat{e}_2(-\hat{P}/2 + \hat{k}_2 + m)\gamma_\mu(\hat{P}/2 - \hat{k}_1 + m)\hat{e}_1 \\ &\times ((k_1 + k_2)p) + \ldots\}u_Q(P/2), \qquad (2) \\ j_\mu &= \bar{u}_q(p_1)\gamma_\mu u_q(-p_2), \qquad (3) \end{aligned}$$

where $d_{abc}(a, b, c = 1, 2, \ldots, N_c^2 - 1)$ are symmetrical constants of $SU(N_c), t_c$ - generators of $SU(N_c)$ group normalized as $Sp(t_a t_b) = \delta_{ab}/2, \psi(0)$ - nonrelativistic wave function of quarkonium in the coordinate space, g_{st} – color charge, $P, p_i = (\varepsilon_i, \vec{p_i}), k_i = (\omega_i, \vec{k_i}), i = 1, 2$ are 4-momenta of the quarkonium, final quark and antiquark and gluons correspondingly, $p = p_1 +$

$p_2, k = k_1 + k_2$, m - mass of the initial quark, u_Q, u_q – bispinors of the initial and final quarks. The dots in (2) imply five permutations of pairs $(\hat{e}_1; k_1), (\hat{e}_2; k_2), (\gamma_\mu; p)$.

The corresponding to (1) – (3) differential probability averaged over three spin states of initial orthoquarkonium and summed over polarizations and colors of final particles may be presented in the form:

$$dW = \frac{F_c}{6\pi^4} \, \frac{\alpha_s^4 \mid \psi(0) \mid^2 Q_{\mu\nu} G_{\mu\nu}}{\left[p^2 (kp)(Pk_1)(Pk_2)\right]^2} \;\; \delta(P - p - k) \, \frac{d\vec{p_1} d\vec{p_2} d\vec{k_1} d\vec{k_2}}{\varepsilon_1 \varepsilon_2 \omega_1 \omega_2}, \qquad (4)$$

where $F_c = (N_c^2 - 4)(N_c^2 - 1)/(32N_c^2)$ is a color factor of $SU(N_c)$ – group, for $SU(3) F_c = 5/36, \alpha_s = g_{st}^2/4\pi$ is the strong coupling constant and

$$Q_{\mu\nu} = \frac{1}{4} \sum_{pol} j_\mu j_\nu^* = p_{1\mu} p_{2\nu} + p_{1\nu} p_{2\mu} - \frac{p^2}{2} g_{\mu\nu}, \qquad (5)$$

$$\begin{aligned} G_{\mu\nu} &= \frac{1}{8m^2} \sum_{pol} J_\mu J_\nu^* = (p^2 g_{\mu\nu} - p_\mu p_\nu) \Big\{ (k_1 k_2)^2 \Big[\frac{(4m^2 - k^2)^2}{8m^2} - p^2 \Big] \\ &+ (pk_1)(pk_2) \Big[2(k_1 k_2) - \frac{(pk_1)(pk_2)}{2m^2} \Big] \Big\} \qquad (6) \\ &- (4m^2 - k^2) l_\mu l_\nu - (k_1 k_2)^2 (T_{\mu\nu}^{11} + T_{\mu\nu}^{22}) \\ &- \frac{1}{2} (4m^2 - k^2) \Big[(k_1 k_2) + \frac{(Pk_1)(Pk_2)}{2m^2} \Big] T_{\mu\nu}^{12}, \end{aligned}$$

with

$$\begin{aligned} l_\mu &= (pk_1) k_{2\mu} - (pk_2) k_{1\mu}, \\ T_{\mu\nu}^{ij} &= 2(pk_i)(pk_j) g_{\mu\nu} + p^2 (k_{i\mu} k_{j\nu} + k_{j\mu} k_{i\nu}) \\ &- (pk_i)(p_\mu k_{j\nu} + p_\nu k_{j\mu}) - (pk_j)(p_\mu k_{i\nu} + p_\nu k_{i\mu}). \end{aligned}$$

The expressions for $Q_{\mu\nu}$ and $G_{\mu\nu}$ obtained by us have the structure such as that of the tensors $L_{\mu\nu}, H_{\mu\nu}$ of Ref.5.

Integrating (4) over quark and antiquark momenta we obtain the probability distribution in the energies $x_i = \omega_i / m$ of the gluons and angle ϑ_g between their momenta in the rest frame of the quarkonium:

$$\frac{d^3W}{dx_1dx_2d\cos\vartheta_g} = F_c\ \frac{\alpha_s^4 \mid \psi(0)\mid^2}{36\pi m^2}\ \frac{F_g}{\eta_g\xi_g^2}\Big(1+\frac{2\mu^2}{\eta_g}\Big)\sqrt{1-\frac{4\mu^2}{\eta_g}}, \tag{7}$$

$$\begin{aligned}
F_g &= 8x_1x_2[12(1+\cos^2\vartheta_g) - 8(x_1+x_2)(1-\cos\vartheta_g+\cos^2\vartheta_g) \\
&+ 4(1-\cos\vartheta_g)[2(x_1+x_2)^2 - x_1x_2(1-\cos\vartheta_g-\cos^2\vartheta_g)] \\
&- 8x_1x_2(x_1+x_2)(1-\cos^2\vartheta_g) + x_1^2x_2^2(1-\cos\vartheta_g)^3(3-\cos\vartheta_g)], \\
\eta_g &= (P-k)^2/m^2 = 4(1-x_1-x_2) + 2x_1x_2(1-\cos\vartheta_g), \\
\xi_g &= ((Pk)-k^2)/m^2 = 2(x_1+x_2) - 2x_1x_2(1-\cos\vartheta_g),
\end{aligned}$$

where $\mu = m_q/m$ is the mass ratio of final and initial quarks, $P = (2m, \vec{0})$.

Integrating $G_{\mu\nu}$ (6) over the momenta of the gluons we find:

$$\begin{aligned}
F_{\mu\nu} &\equiv \int \frac{G_{\mu\nu}}{(Pk_1)^2(Pk_2)^2}\ \delta(k-k_1-k_2)\frac{d\vec{k}_1 d\vec{k}_2}{\omega_1\omega_2} \\
&= \frac{\pi}{2zv^4}\Big\{ - f_1\Big(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\Big) + f_2\ \frac{q_\mu q_\nu}{p^2m^4}\Big\},
\end{aligned} \tag{8}$$

where

$$\begin{aligned}
q_\mu &= p^2P_\mu - (Pp)p_\mu, \\
f_i &= g_i + h_i\frac{1-v^2}{2v}\ln\frac{1+v}{1-v}, \\
g_1 &= v^2z[-v^8z^4 + 4v^6z^2(z^2+1) - 2v^4(3z^4-8z^3+14z^2-24z+32) \\
&+ 4v^2(x^4-8z^3+19z^2-24z+32) - z(z^3-16z^2+52z-48)], \\
h_1 &= v^2z[-v^8z^4 + 2v^6z^2(3z^2+2) - 4v^4(2z^4-4z^3-3z^2-12z+16) \\
&+ 2v^2(z^4-2z^2-32) + z(z^3-16z^2+52z-48)], \\
g_2 &= -v^8z^3 + 20v^6z + 2v^4(3z^3-24z^2+2z-8) \\
&- 4v^2(2z^3-24z^2+45z-24) + 3(z^3-16z^2+52z-48), \\
h_2 &= -v^8z^3 + 2v^6z(z^2+10) - 4v^4(z^3-4z^2+z+4) \\
&+ 2v^2z(3z^2-32z+38) - 3(z^3-16z^2+52z-48), \\
z &= \frac{(Pk)}{2m^2}, \qquad v = \sqrt{1-\frac{4m^2k^2}{(Pk)^2}}.
\end{aligned}$$

Here z and v are the energy and the center-of-mass velocity of the gluon pair in the rest frame of quarkonium.

Using (4), (5) and (8) we obtain the probability distribution in the energies $y_i = \varepsilon_i/m$ of quarks and angle ϑ_q between their momenta in the rest frame of the quarkonium:

$$\frac{d^3W}{dy_1 dy_2 d\cos\vartheta_q} = F_c \, \frac{\alpha_s^4|\psi(0)|^2}{36\pi m^2} \, \frac{F_q}{\eta_q^2\xi_q^2}\sqrt{(y_1^2-\mu^2)(y_2^2-\mu^2)}, \tag{9}$$

where

$$\begin{aligned}
F_q &= \frac{12}{\pi}\frac{F_{\mu\nu}Q_{\mu\nu}}{m^2} = \frac{6}{zv^4}[f_1(\eta_q+2\mu^2)+2f_2\eta_q(4y_1y_2-\eta_q)], \\
\eta_q &= p^2/m^2 = 2\Big[\mu^2+y_1y_2-\cos\vartheta_g\sqrt{(y_1^2-\mu^2)(y_2^2-\mu^2)}\Big], \\
\xi_q &= ((Pp)-p^2)/m^2 = 2(y_1+y_2)-\eta_q, \\
z &= 2-y_1-y_2, \qquad v = \frac{\sqrt{(y_1+y_2)^2-\eta_q}}{z},
\end{aligned}$$

It's convenient for the further analysis to use the distributions (7), (9) normalized as

$$\begin{aligned}
f_g(x_1,x_2,\cos\vartheta_g) &= \frac{1}{\alpha_s W_{3g}} \quad \frac{d^3W}{dx_1dx_2d\cos\vartheta_g}, \\
f_q(y_1,y_2,\cos\vartheta_q) &= \frac{1}{\alpha_s W_{3g}} \quad \frac{d^3W}{dy_1dy_2d\cos\vartheta_q}, \\
W_{3g} &= 2F_c \quad \frac{16(\pi^2-9)}{9} \quad \frac{\alpha_s^3 \mid \psi(0) \mid^2}{m^2},
\end{aligned} \tag{10}$$

where W_{3g} – the three-gluonic decay probability of orthoquarkonium.

The analysis of the quark distribution f_q shows that the probability of the production of light q$\bar{\text{q}}$– pair and two gluons can significantly increase as the angle between quarks decreases. This collinear effect has a simple origin – the decrease of the denominator in the propagator of the virtual gluon as the angle between light quarks decreases and essentialy depends on the mass ratio of the final and initial quarks. As an example we present the quark

distribution f_q as a function of ϑ_q and μ at $y_1 = y_2 = y = 0.4$ in Fig.2. Here one can see that the quark collinear effect becomes significant at $\mu \leq 0.15$ and $\vartheta_q \leq 30°$.

Taking the masses of the quarks into account we conclude that the quark collinear effect takes place in all the quark-gluon decays of the orthocharmonium. As concerning the quark-gluon decay of the orthobottomonium the quark collinear effect takes place in the decays with production of uū-, dd̄- and ss̄- pairs only but is absent in the decay with production of more heavy cc̄- pair because of the rather great mass ratio $\mu = m_c/m_b \sim 0.3$ (see the peak at small angles in Fig.3 and its absence in Fig.4). At $y \geq 0.6$ the region of the small angles becomes kinematicaly forbidden and the collinear effect dissapeares.

By the production of hard ($y \geq 0.8$) qq̄- pair with large angle between q- and q̄- quarks the energy of two accompanying gluons is small and the soft gluon enhancement of the qq̄- production occurs in this case (see the quark curves c in Fig.3,4).

The analysis shows that in contrast with the quark distribution f_q there is no collinear enhancement in the gluon distribution f_g, there is only the soft quark enhansement of the production of the hard gluons accompanied by the light qq̄- pair (see gluon curve c in Fig.3).

In conclution we resume the main results of the work:

1. The decays $^3S_1(Q\bar{Q}) \to q\bar{q}gg$ of heavy orthoquarkonia are considered and the probability distributions in the gluon and in the quark variables are found.

2. The enhancement of these decays caused by the collinearization of the final quarks is discussed in the mass ratio of the final and initial quarks dependence. It is shown that this effect is to take place at the angles $\leq 30°$ in all the decays $^3S_1(c\bar{c}) \to q\bar{q}gg$ of charmonium, in the decays $^3S_1(b\bar{b}) \to q\bar{q}gg$ of bottomonium with the production of uū–, dd̄– and ss̄– pairs but to be absent in the decay $^3S_1(b\bar{b}) \to c\bar{c}gg$.

3. The collinear enhancement in the gluon distribution is shown to be absent in these decays.

4. The production of hard quarks (gluons) accompanied by two soft gluons (quarks) is shown to be enhanced in these decays too.

The quark and gluon distributions obtained by us and their peculiarities discussed here may be useful for the experimental searches and investigations of the four-jet events from the decays of heavy orthoquarkonia.

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Figure captions

1. Diagrams of the decay ${}^3S_1(Q\bar{Q}) \to q\bar{q}gg$ in the tree approximation.
2. The quark distribution f_q as a function of ϑ_q and mass ratio μ at $y_1 = y_2 = y = 0.4$.
3. Angle distributions of gluons at $x_1 = x_2 = x$ and quarks at $y_1 = y_2 = y$ in the decay ${}^3S_1(b\bar{b}) \to s\bar{s}gg$: a) $x = 0.4$ or $y = 0.4$; b) $x = 0.6$ or $y = 0.6$; c) $x = 0.8$ or $y = 0.8$.
4. Angle distributions of gluons at $x_1 = x_2 = x$ and quarks at $y_1 = y_2 = y$ in the decay ${}^3S_1(b\bar{b}) \to c\bar{c}gg$: a) $x = 0.4$ or $y = 0.4$; b) $x = 0.6$ or $y = 0.6$; c) $y = 0.8$.